An Operator Equation and Relativistic Alterations in the Time for Radioactive Decay

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ABSTRACT. In this paper, using concepts from the nonstandard physical world, the linear effect line element is derived. Previously, this line element was employed to obtain, with the exception of radioactive decay, various experimentally verified special theory relativistic alterations in physical measures. This line element is now used to derive, by means of separation of variables, an expression that predicts the same increase in the decay time for radioactive material as that predicted by the Einstein time dilation assumption. This indicates that such an increase in lifetime can be attributed to an interaction of the radioactive material with a nonstandard photon-particle medium so as to maintain hyperbolic velocity-space behavior.

Key Words and Phrases. Special relativity, separation of variables, radioactive decay, time dilation, nonstandard analysis.
1992 AMS Subject Classifications. 83A05, 03H10.

1. Introduction.

In [7], a specific operator equation related to a partial differential equation, the Schwarzschild and linear effect line elements, and the concept of separation of variables are used to derive various relativistic alteration expressions. As discussed in [6], the measurable aspects of the special theory of relativity need not be related to the so-called general time dilation or length contraction concepts. It is conjectured that all of the special theory predicted alterations in measured quantities can be obtained by means of electromagnetic propagation model. The quantities

* Partially funded by a grant from the United States Naval Academy Research Council. This version corrects notational errors that appear in the published version [8], adds new material as well as presenting a simple and correct derivation for the alteration in the rate of radioactive decay.

must be measured by infinitesimal light-clocks or an equivalent approximating device and, in order to avoid the model theoretical error of generalization, no other instrumentation is allowed.

In [7], with the exception of relativistic alterations in the lifetime of radioactive material, all of the major special theory relativistic alterations in measured quantities are obtained from the linear effect line element. What is derived first is a statement relating infinitesimal time measurements, where these measurements are interpreted as changes in the number of counts or "ticks" that occur within a measuring infinitesimal light-clock. The use of infinitesimal light-clocks to obtain the experimentally verified alterations indirectly implies that such alternations may be caused by (emis) - interactions within a nonstandard photon-particle medium (NSPPM) using a photon particle behavioral model.

The main purpose of this paper is to present a derivation of the linear effect line element using a rather simple infinitesimal NSPPM process and to use the methods in [7] to predict the experimentally verified [1] relativistic alteration in radioactive decay time.

2. The Linear Effect Line Element.

In what follows, we are considering a standard set-theoretic superstructure $\mathcal{M} = \langle \mathbb{R}, \in, = \rangle$ with ground set the real numbers $\mathbb{R}$ and this structure is embedded into a nonstandard elementary extension ${}^*\mathcal{M} = \langle {}^*\mathbb{R}, \in, = \rangle$ that is an enlargement [10]. In [5a, b], the chronotopic interval (i.e. proper time interval) is derived and yields the following interpretation. An infinitesimal light-clock is in linear and uniform motion, and L is an infinitesimal number representing the length of the arm. The infinitesimal light-clock has "ticked" $\Pi_m \in \mathbb{N}_\infty$ times, where $\mathbb{N}_\infty$ is the set of Robinson infinite natural numbers. The apparent physical world linear distance traversed by the electromagnetic radiation in its to-and-fro motion within the moving infinitesimal light-clock is $\mathtt{st}(2L\Pi_m)$.

At a space-time point, there are four infinitesimal light-clocks. These four infinitesimal light-clocks are used to obtain Einstein measurements [9], which are obtained by the "radar" method for determining the relative velocity of a moving light-clock, the distance and "time" measurements at a point m in motion relative to a point s. The radar method is used since retaining this mode of measurement the twin anomaly is eliminated. This anomaly is not eliminated when other derivations, independent from the mode of measurement, are used. Further, the GR approach does not eliminate this anomaly under reasonable conditions and, especially, when

three objects are used [9]. In general, Einstein time, distance and velocity are $t_E = (1/2)(t_3 + t_1)$, $r_E = (1/2)c(t_3 - t_1)$, $v_E = r_E/t_E$, when defined. The t_1 is the moment of time at the s-point when the light-pulse is emitted and t_3 the time when it is received back at the s-point after being "reflected" from the m-point. Notice that when $r_E = 0$, then $t_E = t^3$.

Three infinitesimal light-clocks that correspond to a Cartesian coordinate system are used to measure local distance at the s-point and the fourth infinitesimal light-clock is used to measure time at the s-point. In particular, the superscript and subscript s represents local measurements about the s-point, using various devices, for laboratory standards (i.e. standard behavior), and infinitesimal light-clocks or approximating devices such as the atomic-clocks. [Due to their construction, atomic clocks are affected by relativistic motion and gravitational fields approximately as the infinitesimal light-clock's counts are affected.] Superscript or subscript m indicates local measurements at the m-point for an entity considered at the m-point in motion relative to the s-point. The local time measure at the m-point is the Einstein time, via the radar method, as registered at the s-point. The relative velocity and distance between the points are also the Einstein measurements. These measurements are used to investigate m-point behavior. To determine how physical behavior is being altered, the m-point and s-point measurements are compared.

Let $\omega \in \mathbb{N}_\infty$. Since the sequence $S_n = n \to \infty$, then as proved in [4, p. 100] for each $r \in \mathbb{R}$, there exists an $x \in \{m/\omega \mid (m \in {}^*\mathbf{Z}) \wedge (|m| < \omega^2)\}$ such that $x \approx r$. As discussed in [6] and due to this result, the use of infinitesimal light-clocks allows for time and length measurements to be considered as varying over intervals of real numbers. This might be expressed by saying that they can take on a "continuum" of values. Thus standard analysis can be applied.

Let (x^s, y^s, z^s, t^s) be Cartesian coordinates that are the standard part of s-point infinitesimal light-clock measurements for locally stationary points. Let (x^m, y^m, z^m, t^m) be Cartesian coordinates that are the standard part of m-point infinitesimal light-clock measurements as registered at the s-point via Einstein measures since this application is for special relativity. At the s-point the infinitesimal light-clock count ("ticks") are used to obtain time intervals. At the moment when the s-point infinitesimal light-clock has count number Π'_s, a signal is sent to the m-point and its reception, as measured by Einstein time at the s-point, is at count number Π'_m. A second single is sent from the s-point to the m-point and the two count numbers are Π''_s, Π''_m. The infinitesimal light-clock measured interval is $\Pi_s = \Pi''_s - \Pi'_s$. The length of s-point time interval is the real number $\mathtt{st}(u\Pi_s) = \Delta t^s$.

For the m-point, even when using Einstein measures, the length of the time interval can be expressed as $\Pi_m = \Pi''_m - \Pi'_m$, and has the real value $\mathtt{st}(u\Pi_m)$. Using these count numbers, the expression derived in [5a,b] is

$$(\mathtt{st}(L\Pi_m))^2 = (\Delta t^s)^2 c^2 - ((\Delta x^s)^2 + (\Delta y^s)^2 + (\Delta z^s)^2), \tag{2.1}$$

where c is the to-and-fro local measurement at the s-point for the velocity of electromagnetic radiation. Physical measurements can be considered as absolute with respect to a third position fixed in the NSPPM. In this case, the three relative velocities are related by the velocity composition expression [5b, p. 48], where the NSPPM velocities follow simple linear Galilean rules.

From a quantum-physical viewpoint, certain photon interactions can be viewed as mimicking the basic to-and-fro light-clock process. Hence, light-clocks are the best form of timekeeping that satisfies the rules for infinitesimalizing. This allows for the infinitesimalizing of expression (2.1) and yields

$$dS^2 = (dt^s)^2 c^2 - ((dx^s)^2 + (dy^s)^2 + (dz^s)^2). \tag{2.2}$$

Equation (2.2) are infinitesimal light-clock measures. This correspond to the classical approach, when ds is restricted to infinitesimal light-clock counts. Expression (2.2) is that which is used when (2.1) is extended so as to include nonuniform behavior. We often write $(dr^s)^2 = (dx^s)^2 + (dy^s)^2 + (dz^s)^2$.

Expression (2.2) relates behavior of infinitesimal light-clock counts but does not indicate that there might be a cause for such behavior. Distinct from the classical approach to the special theory, a cause can be postulated relative to the behavior of the NSPPM. In what follows, the timing infinitesimal light-clocks are used as an analogue model to investigate how the NSPPM behavior is altered by a process P. It is shown [5, pp. 50-54] that, for special relativity, global NSPPM light propagation within our physical world using radar measured velocities is related to an NSPPM hyperbolic velocity-space. The P-process within the NSPPM has the effect that the Einstein measurements for relative velocity v are directly related to an NSPPM velocity $w = (c/2)\ln((1 + v^2/c^2)/(1 - v^2/c^2))$ [5b, p. 51. Further, the P-process is related to motion viewed from points in Euclidean space. Suppose that the counts of these measuring infinitesimal light-clocks are affected by this P-process. We seek a relationship $\phi(dx^m, dy^m, dz^m, dt^m)$ or $\Phi(dr^m, dt^m)$ between altered counts as it is obtained by measuring infinitesimal light-clocks so that $L\Pi_m = \phi(dx^m, dy^m, dz^m, dt^m)$ or $L\Pi_m = \Phi(dr^m, dt^m)$.

Following [5, 6], let v, d and c behave within a monadic neighborhood as if they are constant with respect to P. Since behavior in an infinitesimal neighborhood is supposed to be simple behavior, then the simple Galilean velocity-distance law holds. For photon behavior with a moving source velocity $v+d$,

$$((v+d)+c)dt^s = (v+d)dt^s + cdt^s = dR^s + dT^s,$$

$$dT^s = cdt^s,\ dR^s = (v+d)dt^s,\ dt^s \neq 0,\ \frac{dR^s}{dT^s} = \frac{v+d}{c},\ . \tag{2.3}$$

Suppose that, due to P, a smooth microeffect [3] alters the infinitesimal light-clock counts. For $(dr^m)^2 = (dx^m)^2 + (dy^m)^2 + (dz^m)^2$, this alteration is characterized by the infinitesimal linear transformation (A): $dr^s = (1-\alpha\beta)dr^m - \alpha dT^m$, (B): $dT^s = \beta dr^m + dT^m,\ dT^m = cdt^m$, where $\alpha,\ \beta$ are to be determined. Since the effects are to be observed in the physical world, $\alpha,\ \beta$ have standard values.

Substituting (A) and (B) into (2.2) yields

$$dS^2 = (1-\alpha^2)(dT^m)^2 + 2(\alpha + \beta(1-\alpha^2))dr^m dT^m +$$

$$(\beta^2 - (1-\alpha\beta)^2)(dr^m)^2. \tag{2.4}$$

The simplest real world aspect of time interval measurement that assumes that timing counts can be added or subtracted is transferred to a monadic neighborhood and requires dT^m to take on positive or negative infinitesimal values. As done for space-time, suppose that the P-process is symmetric with respect to the past and future sense of a time variable. This implies that dS^2 is unaltered when dt^m is replaced by $-dt^m$. This implies that the transformation needs to be restricted so that $2(\alpha + \beta(1-\alpha^2)) = 0$. For simplicity of calculation, let $\alpha = -\sqrt{1-\eta}$. Hence, $\beta = \sqrt{1-\eta}/\eta$. Substituting into (A) and (B) yields

$$dr^s = \frac{1}{\eta}dr^m + \sqrt{1-\eta}\,dT^m$$

$$dT^s = \frac{\sqrt{1-\eta}}{\eta}dr^m + dT^m. \tag{2.5}$$

Combining both equations in (2.5) produces

$$\frac{dr^s}{dT^s} = \frac{\frac{1}{\eta}\frac{dr^m}{dT^m} + \sqrt{1-\eta}}{\frac{\sqrt{1-\eta}}{\eta}\frac{dr^m}{dT^m} + 1}. \tag{2.6}$$

The x^m, y^m, z^m are not dependent upon t^m. Hence the (total) derivative $dr^m/dT^m = 0$. Using (2.6), $dr^s/dT^s = (v+d)/c = \sqrt{1-\eta}$ or $\eta = 1-(v+d)^2/c^2 = \lambda \neq 0$. We note that using $\alpha = \sqrt{1-\eta}$ yields the contradiction $(v+d)/c < 0$ for the case being considered that $0 \leq v+d < c$. By substituting η into (2.5) and then (2.5) into (2.2) (see (2.4)), we have, where $dT^m = cdt^m$, the linear effect line element

$$(cdt^s)^2 - (dr^s)^2 = dS^2 = \lambda(cdt^m)^2 - (1/\lambda)(dr^m)^2. \tag{2.7}$$

This linear effect line element yields a special theory line element if $v = v_E$ (the Einstein measure of the relative velocity) and correlates special theory effects to a describable P-process. This process includes the appropriate linear Galilean rules for uniform motion that in the NSPPM yield the hyperbolic velocity-space for the observed physical-world. Equation (2.7) represents the necessary alteration in the infinitesimal light-clock counts when they are affected by the P-process. Since the line element is a relation between infinitesimal quantities, the expression "linear effect" is not intended to imply that the (standard) path of motion of the analogue infinitesimal light-clock is necessarily linear.

[Transforming (2.2) via the spherical coordinate transformations, a similar argument for a static P-process yields a general radial effect line element. By selecting various "potential" velocities v and d (i.e. whatever v and d are they have velocity units of measure), this general radial line element is transformed, at the least, into the Schwarzschild, Schwarzschild with cosmological constant, de Sitter, and even the Robertson-Walker line elements. Indeed, with appropriate v and $d = 0$, (2.7) is the approximating Newtonian line element. Moreover, (2.7) applies directly to regions where gravitational potentials are constant or approximately so. For gravitational potentials, Einstein measurements are not used.]

Following the usual practice for decay purposes, it is assumed that the atomic structure is momentary at rest when decay occurs. This is modeled by letting $dr^s = 0$ in (2.2) and $dr^m = 0$ in (2.7). Hence we have that (*) $\gamma dt^m = dt^s$, where $\gamma = \sqrt{\lambda} \neq 0$. This does not indicate a change in the concept of time. As implied in [6] and shown in [5], for zeroed infinitesimal light-clocks, $dt^m = u\Delta\Pi_m$, $dt^s = u\Delta\Pi_s$. The NSP-world time unit u is not altered, but the infinite count number $\Delta\Pi_s$ has been altered. Using this analogue approach, if the verified alterations in decay rates can be predicted, then this would indicate that radioactive decay itself is related to an (emis) effect.

3. The Derivation of the Decay Prediction.

The method of separation of variables as used in [7] is the consistent underlying procedure used to predict the verified alteration in decay time. Unfortunately, in [7] a confusing typographical error occurs in expressions (3.2) and (3.3). The symbols $h(r^s, t^s)$ should be replaced with $h(r^s)$ and $H(r^m, t^m)$ should be replaced with $H(r^m)$.

Let $N(t^s)$ denote a measure for the number of active entities at the light-clock count time t^s and τ_s be the (mean) lifetime. These measures are taken within a laboratory and are used as the standard measures. This is equivalent to saying that they are, from the laboratory viewpoint, not affected by relativistic alterations. The basic statement is that there exists some $\tau \in (0, B]$ such that (*) $(-\tau)dN/dt = N$. Even though the number of active entities is a natural number, this expression can only have meaning if N is differentiable on some time interval. But, since the τ are averages and the number of entities is usually vary large, then such a differentiable function is a satisfactory approximation. Recall that the required operator expression is

$$D(T) = k(\partial/\partial t)(T). \tag{3.1}$$

Let $k = 1$ and $h(r) = 0 \cdot r^2 + 1 = 1$. Then define $T(r, t) = h(r)N(t) = (0 \cdot r^2 + 1)N(t)$, where $r^2 = x^2 + y^2 + z^2$, and let D be the identity map I on $T(r, t)$. Then $D(T(r, t)) = D(h(r))N(t) = D(0 \cdot r^2 + 1)N(t) = 1 \cdot N(t)$ and, in this form, D is considered as only applying to h and it has no effect on $N(t)$. In this required form [5b, p. 62; 7, p. 60], first let $r = r^s$ and $t = t^s$. Further, consider $T(r^m, t^m) = H(r^m)\overline{N}(t^m)$, $H(r^m) = 0 \cdot (r^m)^2 + 1 = 1$. In order to determine whether there is a change in the τ_s, one considers the value $N(t^s) = \overline{N}(t^m)$. This yields the final requirement for T. Notice that t^m is Einstein time as measured from the s-point. This is necessary in that the $v = v_E$, which is a necessary requirement in order to maintain the hyperbolic-velocity space behavior of v.

Applying (3.1) to T and considering the corresponding differential equation (*) and the chain rule, one obtains that there exist a real number τ_s such that

$$\begin{gathered} N(t^s) = (-\tau_s)(d/dt^s)N(t^s) = \\ (-\tau_s)(d/dt^m)\overline{N}(t^m)(d/dt^s)(t^m) = \\ (-\tau_s/\gamma)(d/dt^m)\overline{N}(t^m). \end{gathered} \tag{3.2}$$

And, with respect to m, and for τ_m

$$\overline{N}(t^m) = (-\tau_m)(d/dt^m)\overline{N}(t^m). \quad 3.3)$$

Using (3.2) and (3.3) one obtains that $\tau_m = \tau_s/\gamma$. (In the linear effect line element $d = 0$.) This is one of the well-known expressions for the prediction for the alteration of the decay rates due to relative velocity (that is v_E). The τ_s can always be taken as measured at rest in the laboratory since the relative velocity of the active entities is determined by experimental equipment that is at rest in the laboratory.

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